

Theme

"Think deeply about simple things"

STEM Innovation Academy Unit 2

Subject: Calculus
Unit Title: Derivatives
Grade: 11

Teacher: Ahmed Salama
Duration: 8 weeks

Summary of Unit

In Unit 1 students learned how to determine the slope of a curve at a point and how to measure the rate at which a function changes. Now they have studied limits, they can make these notions precise and see that both are interpretations of the derivative of a function at a point. They then extend this concept from a single point to the derivative function, and they develop rules for finding this derivative function easily, without having to calculate limits directly. These rules are used to find derivatives of most of the common functions reviewed in last unit as well as combinations of them.

The derivative is used to study a wide range of problems in mathematics, science, economics, and medicine. In this unit students will explore and solve these problems including finding solutions to very general equations, calculating the velocity and acceleration of a moving object, describing the path followed by a light ray going from a point in air to a point in water, finding the number of items a manufacturing company should produce in order to maximize its profits, studying the spread of an infectious disease within a given population, and calculating the amount of blood the heart pumps per minute based on how well the lungs are functioning.

Textbook: Thomas' Calculus Early Transcendentals (Fourteenth Edition)
Online resource: MyMathLab www.mymathlab.com

Stage 1 – Desired Results

Essential Questions:

- How can we find the instantaneous rate of change of a particle in motion?
- What is the relationship between the graph of a function and its derivative ?
- What is the relationship between differentiability and continuity?

Enduring Understandings:

- Derivatives can be used to analyze curves and model rates of change by using the difference quotient to find a derivative at a point, applying the concept of derivative to write tangent line equations, identify how a function is changing based on its derivative
- Differentiability implies continuity, identify points where a function may be continuous and not differentiable
- Finding the derivative of a function may require the use of several rule, including rules for; sums, products, quotients, powers.
- The physics concepts of position, velocity and acceleration are related mathematically by the derivative.

Objectives:

After completing the learning of this unit, student will be able to

- define the slope and tangent to a curve at a point, and derivative of a function at a point.
- find a tangent line to the graph of a function

- understand the derivative as a function derived from f by considering the limit at each point x in the domain of f
- apply Alternative Formula for the derivative
- calculate derivatives from the definition, graph the derivative
- understand, prove and apply Positive Integer Power Rule, Derivative Constant Multiple Rule, Derivative Sum Rule, Derivative Product Rule, Derivative Quotient Rule
- derivative of the natural exponential function
- find second and higher-order derivatives
- find instantaneous rates of change
- find a body's velocity at the exact instant t
- differentiate the six basic trigonometric functions
- develop the Chain Rule
- differentiate a composite function
- use Chain rule to extend a differentiable function of x to the Power Chain Rule: $(d/dx)(u^n) = nu^{n-1}(du/dx)$
- apply implicit differentiation to find dy/dx
- use implicit differentiation to find higher derivatives
- develop a rule for differentiating the inverse of a differentiable function, and apply the rule to find the derivative of the natural logarithm function
- understand how all six inverse trigonometric functions are defined
- graph and evaluate all six inverse trigonometric functions
- compute the derivatives of all six inverse trigonometric functions
- understand related rates and solve related rates problems
- understand the meaning of linearization and use the concept to approximate functions.

Standards/Outcomes: Collage Board

- D2C1: Approximate the derivative. Graphically by finding the slope of a tangent line drawn to a curve at a given point. Numerically by using the difference quotient
- D2C2: Find the equation of the tangent line using the definition of derivative
- D2C7: Find derivatives of functions using: Power rule, Product rule, Quotient rule.
- D2C8: Find derivatives of:
 - an implicitly defined equation,
 - composite functions using chain rule
 - exponential and logarithmic functions
 - functions requiring the use of more than one differentiation rule
- D2C9: Find the equation of:
 - A line tangent to the graph of a function at a point
 - A normal line to the graph of a function at a point
- D2C11: Interpret the derivative as a rate of change and varied applied contexts, including velocity, speed, and acceleration.

Unit Math Practice Standards:

- **MP.1** Make sense of problems and persevere in solving them.
- **MP.2** Reason abstractly and quantitatively.
- **MP.3** Construct viable arguments and critique the reasoning of others.
- **MP.4** Model with mathematics.
- **MP.5** Use appropriate tools strategically.
- **MP.6** Attend to precision.
- **MP.7** Look for and make use of structure.
- **MP.8** Look for and express regularity in repeated reasoning.

Stage 2 – Assessment Evidence**Performance Task(s):**

- Task 1: Rate of change to displacement of an object with respect to the time
- Task 2: The derivative of logarithmic of trigonometric function using chain's rule
- Task 3: Creating a table to discuss function concavities

Authentic Experiences:

- Given a curve of polynomial function, find the critical point, study concavity using the first and second derivative
- Given an equation that represent displacement of a flying object find its velocity and acceleration at specific time using first and second derivative
- Showing cylindrical tank is filling with oil, find the change of volume when height change, with respect to time.

Unit Pre-Assessment:

- NJIT Unit Diagnostic Assessment

Presentation: Focus

- Discuss knowledge of limits to derivative of function (Fundamental theorem)
- Using rules to find derivative
- Find derivative of product and quotient of functions
- Find derivative of generated functions (Chain's Rule)
- Apply derivative to our real-life situations (Related Rates)

Summative Assessment:

- NJIT common Midterm Exam II

Extensions (Tier I):

- Enrichment Question (challenging add-ons)
- More applications of findings

Differentiation (Tiers 2 and 3):

- Grouping students (cooperative learning)
- Small group support or re-teaching based on assessment data
- MyMathLab online work support

Stage 3 – Learning Plan

Vocabulary

- Slope of the curve
- Tangent lines
- Derivative
- Differentiation
- Differentiable functions
- Rules of Powers, Multiples, Sums, and Differences
- Natural exponential function
- Second derivative
- Instantaneous rate of change
- Displacement
- Average velocity
- Acceleration
- Jerk
- Simple Harmonic Motion
- Chain Rule
- Implicit Differentiation
- Lenses, Tangent Lines, and Normal Lines
- Derivative Rule for Inverse
- Alternative Derivation
- Inverse Trigonometric Functions
- Rated rates
- Approximating function
- Linearization
- Approximation
- Differential

Learning Material:

Textbook: Thomas' Calculus Early Transcendentals (Fourteenth Edition)

Section 3.1: Tangents and Derivatives at a Point (page 121-124)

Section 3.2: The Derivative as a Function (Page 124 – 134)

Section 3.3: Differentiation Rules (Page 134-144)

Section 3.4: Derivatives as a Function (Page 144-154)

Section 3.5: Derivatives to Trig Functions (Page 154-160)

Section 3.6: The Chain Rule (Page 161-168)

Section 3.7: Continue Chain Rule; Start Implicit Differentiation (Page 169-174)

Section 3.8: Derivatives of Inverse and Log Functions (Page 174-183)

Section 3.9: Inverse Trig Functions (Page 184-191)

Section 3.10: Related Rate (Page 191-199)

Section 3.11: Linearization and Differentials (Page 200-211)